____ MTH 225[M]

(Name: last, first) Oct:15(Thu)

Midterm Examination

Acknowledgement: Some of these exercises are derived from Kenneth Rosen.

A. For each of the following English sentences, state what the sentence means if the "or" is an inclusive "or" (that is, a disjunction) versus an exclusive "or":

1. [2 points] "To take discrete mathematics, you must have taken calculus or a course in computer science."

2. [2 points] "When you buy a new car from Acme Motor Company, you get $2000 back in cash or a 2% car loan."

B. [2 points] Give the contrapositive of the following implication. Use the form, "If . . . , then . . . ."

"If there is a test, then I come to class."

C. 1. [6 points] Complete a truth table as follows for the logic formula $(\neg \mathtt{P} \land \mathtt{R}) \lor (\mathtt{Q} \to \neg \mathtt{R})$. Remember that you need to have a column for each subformula such as $\neg \mathtt{P}$ and $\mathtt{Q} \to \neg \mathtt{R}$ in addition to the columns for P, Q, R, and the final column for the overall formula.

```
 P | Q | R ||
---+---+---++--------------------------------------------------------------
 t | t | t ||
---+---+---++--------------------------------------------------------------
 t | t | f ||
---+---+---++--------------------------------------------------------------
 t | f | t ||
---+---+---++--------------------------------------------------------------
 t | f | f ||
---+---+---++--------------------------------------------------------------
 f | t | t ||
---+---+---++--------------------------------------------------------------
 f | t | f ||
---+---+---++--------------------------------------------------------------
 f | f | t ||
---+---+---++--------------------------------------------------------------
 f | f | f ||
```

2. [2 points] Is the logic formula $(\neg \mathtt{P} \land \mathtt{R}) \lor (\mathtt{Q} \to \neg \mathtt{R})$ a tautology? Explain.

D. [2 points] Consider the following English sentence:

> For any values $\mathtt{n}$ and $\mathtt{d}$, $\mathtt{d|n}$ holds precisely when $\mathtt{n} = \mathtt{d}*\mathtt{q}$ for some value $\mathtt{q}$.

Express that English sentence in logic. Use the predicate symbol $\mathtt{D}(\ldots,\ldots)$ to express the part "$\mathtt{d|n}$".

E. [10 points] (Acknowledgement: This exercise is derived from Kenneth Rosen.)
Let $\mathtt{Q}(\mathtt{x},\mathtt{y})$ represent the statement "$\mathtt{x}+\mathtt{y} = \mathtt{x}-\mathtt{y}$". If the domain for both variables consists of all integers, what are the truth values of the following formulas?

1. $\mathtt{Q}(\mathtt{1},\mathtt{1})$
2. $\mathtt{Q}(\mathtt{2},\mathtt{0})$
3. $\forall \mathtt{y} \mathtt{Q}(\mathtt{1},\mathtt{y})$
   Explain.
4. $\exists \mathtt{x} \mathtt{Q}(\mathtt{x},\mathtt{2})$
   Explain.
5. $\forall \mathtt{y} \neg \mathtt{Q}(\mathtt{0},\mathtt{y})$
   Explain.
6. $\exists \mathtt{x} \neg \mathtt{Q}(\mathtt{x},\mathtt{0})$
   Explain.

F. [6 points] Here are some of the logical simplifications. A "φ" (possibly subscripted) represents an arbitrary logic formula, "$\mathtt{f}$" stands for "$\mathtt{false}$", and "$\mathtt{t}$" stands for "$\mathtt{true}$".

$$\begin{array}{ccccccc}
\mathtt{f} \wedge \varphi & \varphi \wedge \mathtt{f} & \mathtt{t} \wedge \varphi & \varphi \wedge \mathtt{t} & \varphi \wedge \varphi & \varphi \wedge \neg\varphi & \neg\varphi \wedge \varphi \\
\Downarrow & \Downarrow & \Downarrow & \Downarrow & \Downarrow & \Downarrow & \Downarrow \\
\mathtt{f} & \mathtt{f} & \varphi & \varphi & \varphi & \mathtt{f} & \mathtt{f}
\end{array}$$

$$\begin{array}{ccccccc}
\mathtt{f} \vee \varphi & \varphi \vee \mathtt{f} & \mathtt{t} \vee \varphi & \varphi \vee \mathtt{t} & \varphi \vee \varphi & \varphi \vee \neg\varphi & \neg\varphi \vee \varphi \\
\Downarrow & \Downarrow & \Downarrow & \Downarrow & \Downarrow & \Downarrow & \Downarrow \\
\varphi & \varphi & \mathtt{t} & \mathtt{t} & \varphi & \mathtt{t} & \mathtt{t}
\end{array}$$

$$\begin{array}{cccccccc}
\mathtt{f} \rightarrow \varphi & \varphi \rightarrow \mathtt{f} & \mathtt{t} \rightarrow \varphi & \varphi \rightarrow \mathtt{t} & \varphi \rightarrow \varphi & \varphi \rightarrow \neg\varphi & \neg\varphi \rightarrow \varphi & \neg\varphi_1 \rightarrow \neg\varphi_2 \\
\Downarrow & \Downarrow & \Downarrow & \Downarrow & \Downarrow & \Downarrow & \Downarrow & \Downarrow \\
\mathtt{t} & \neg\varphi & \varphi & \mathtt{t} & \mathtt{t} & \varphi & \neg\varphi & \varphi_2 \rightarrow \varphi_1
\end{array}$$

$$\begin{array}{cccccc}
\neg\mathtt{f} & \neg\mathtt{t} & \neg\neg\varphi & \neg(\neg\varphi_1 \wedge \neg\varphi_2) & \neg(\neg\varphi_1 \vee \neg\varphi_2) & \neg(\varphi_1 \rightarrow \neg\varphi_2) \\
\Downarrow & \Downarrow & \Downarrow & \Downarrow & \Downarrow & \Downarrow \\
\mathtt{t} & \mathtt{f} & \varphi & \varphi_1 \vee \varphi_2 & \varphi_1 \wedge \varphi_2 & \varphi_1 \wedge \varphi_2
\end{array}$$

Use those simplifications — *and no other transformations* — to simplify the following formula as much as possible. (Note: your result here may be different from the result you could get from `ProofBuilder`.) Show the steps of simplification, including drawing a box around each subpart before you simplify it and also circling each simplification here (above) that you use. (Don't worry about trying to show exactly when you use each simplification: I should be able to figure it out.) You can do multiple simplifications at one time, but only if the multiple boxes that you need to draw for them do not overlap. And again, use the simplifications listed here (above), *NO OTHER TRANSFORMATIONS!*

$$[\neg \mathtt{P} \lor (\mathtt{t} \land \neg \mathtt{Q})\ ] \ \rightarrow\ (\mathtt{R} \land \mathtt{f})$$

G. [3 points] Starting with the following formula, apply logical equivalences to produce an equivalent formula where negations are applied to only predicates (that is, where no negation is outside a quantifier or an expression whose operator is a logical connective).

$$\neg\forall \mathtt{y} \exists \mathtt{x}[\mathtt{P}(\mathtt{x},\mathtt{y}) \lor \mathtt{Q}(\mathtt{x},\mathtt{y})]$$

H. [10 points] (Acknowledgement: This exercise is derived from Kenneth Rosen.) Write a proof of the following statement:

If $\mathtt{n}$ is even, then $\mathtt{n}^2$ is even.

You'll need to use the following 'presupposition':

```
// DEFINITION 1
```

$$\forall \mathtt{x}\Big(\mathtt{x} \text{ is even \ means } \exists \mathtt{y}[\mathtt{x} = 2\mathtt{y}]\Big)$$

While you're writing this proof, you can use two columns, with your main work in the right column and suppositions in the left column as in `ProofBuilder`; or alternatively you can write this proof in some other say more 'flowing' style as in our textbook; it's your choice. Just try to be as clear and thorough as you can here.

Total Score: / 45