Rosen page267_example1

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Proof of the following formula:

(∀n>0)[(∑ i for i=1 to n) = n(n+1)/2]

(#)	Comments	Suppositions and derivations	Theorem and subgoals
		We presuppose the following:	
(1)		(∀e,v,x)[(∑ e for v=x to x) = e@[v := x]]	
(2)		(∀x,y,e,v) [(∑ e for v=x to y+1) = (∑ e for v=x to y) + e@[v := y + 1]]	
(3)		(∀x,y,z)[x/y + z = (x + yz)/y]	
(4)		(∀x,y,z)[x·z + y·z = z·(x+y)]	
			We start working with the formula being proved as follows:
(5)			(∀n>0)[(∑ i for i=1 to n) = n(n+1)/2]
			Well, invoking stepwise induction, we should prove the following:
(6)			(∑ i for i=1 to 1) = 1(1 + 1)/2 ∧ (∀n) ((∑ i for i=1 to n) = n(n + 1)/2 ∧ n > 0 ⇒ (∑ i for i=1 to (n + 1)) = (n + 1)((n + 1) + 1)/2)
			≡ by simplifying
(7)			(∑ i for i=1 to 1) = 1 ∧ (∀n) ((∑ i for i=1 to n) = n(n + 1)/2 ∧ n > 0 ⇒ (∑ i for i=1 to (n + 1)) = (n + 1)(n + 2)/2)
			We'll handle that formula's subparts in separate cases (below):
			Case 1:
(8)	*basis step*		(∑ i for i=1 to 1) = 1
			≡ by (1) with "e := i, v := i, x := 1"
(9)			[i@[i := 1]] = 1
			≡ by simplifying
(10)			true
			That concludes the proof for this case.
			Case 2:

(11)	*inductive step*		(∀n) ((∑ i for i=1 to n) = n(n + 1)/2 ∧ n > 0 ⇒ (∑ i for i=1 to (n + 1)) = (n + 1)(n + 2)/2)
			Well, let an arbitrary value "k" be given; then we should prove the following:
(12)			(∑ i for i=1 to k) = k(k + 1)/2 ∧ k > 0 ⇒ (∑ i for i=1 to (k + 1)) = (k + 1)(k + 2)/2
		We'll assume that formula's antecedents:	
(13)		(∑ i for i=1 to k) = k(k + 1)/2	
		and also:	
(14)	*[unused]*	k > 0	
			and we'll work on proving the consequent:
(15)			(∑ i for i=1 to (k + 1)) = (k + 1)(k + 2)/2
			We'll work on transforming the left-hand side of formula (15) to the right-hand side as follows:
(16)			∑ i for i=1 to (k + 1)
			= by (2) with "e := i, v := i, x := 1, y := k"
(17)			(∑ i for i=1 to k) + i@[i := k + 1]
			= by simplifying
(18)			(∑ i for i=1 to k) + (k + 1)
			= by (13)
(19)			(k(k + 1)/2) + (k + 1)
			= by (3) with "x := k(k + 1), y := 2, z := k+1"
(20)			([k(k + 1)] + 2(k + 1))/2
			= by (4) with "x := k, z := k+1, y := 2"
(21)			[(k + 1)·(k + 2)]/2
			And that satisfies our earlier goal (15); i.e. we have:
(22)			true
			That concludes this part of the proof.
			Thus, the theorem that was given is true in all cases.

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