Gersting page094_example9

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Proof of the following formula:

odd(xy) ↔ odd(x) ∧ odd(y)

(#)		Suppositions and derivations	Theorem and subgoals
		We presuppose the following:	
(1)		(∀z)[odd(z)' ↔ even(z)]	
(2)		(∀z)[even(z) ↔ (∃k)(z = 2k)]	
(3)		(∀a,b,c)[(ab)c = a(bc)]	
(4)		(∀z)[odd(z) ↔ (∃k)(z = 2k + 1)]	
(5)		(∀a,b,c,d)[(a+b)(c+d) = ac + bc + ad + bd]	
(6)		4 = 2·2	
(7)		(∀a,b,c,d)[ab + ac + ad = a(b+c+d)]	
			We start working with the formula being proved as follows:
(8)			odd(xy) ↔ odd(x) ∧ odd(y)
			≡ by rewriting the form "$\varphi_1 \leftrightarrow \varphi_2$" to "$(\varphi_1 \rightarrow \varphi_2) \land (\varphi_2 \rightarrow \varphi_1)$"
(9)			(odd(xy) → odd(x) ∧ odd(y)) ∧ (odd(x) ∧ odd(y) → odd(xy))
			We'll handle that formula's subparts in separate cases (below):
			Case 1:
(10)			odd(xy) → odd(x) ∧ odd(y)

		≡ by rewriting the form "$\varphi_1 \to \varphi_2$" to "$\varphi_2' \to \varphi_1'$"
(11)		(odd(x) ∧ odd(y))´ → [odd(xy)]´
		≡ by rewriting the form "$(\varphi_1 \wedge \varphi_2)'$" to "$\varphi_1' \vee \varphi_2'$"
(12)		[odd(x)]´ ∨ [odd(y)]´ → [odd(xy)]´
		≡ by (1) with "z := x"
(13)		[even(x)] ∨ [odd(y)]´ → [odd(xy)]´
		≡ by (1) with "z := y"
(14)		[even(x)] ∨ [even(y)] → [odd(xy)]´
		≡ by (1) with "z := xy"
(15)		[even(x)] ∨ [even(y)] → [even(xy)]
	We'll assume that formula's antecedent:	
(16)	[even(x)] ∨ [even(y)]	
		and we'll work on proving the consequent:
(17)		even(xy)
		≡ by (2) with "z := xy"
(18)		(∃k)((xy) = 2k)
		Removing that variable quantification clarifies that to prove that formula, it will suffice to find a value for the variable.
(19)		(xy) = 2k
	We'll handle formula (16)'s subparts in separate cases (below):	
	Case 1a:	
(20)	even(x)	
	≡ by (2) with "z := x"	
(21)	(∃k)(x = 2k)	

		That formula indicates there exists some value say "m_1" satisfying that quantified formula, i.e.:	
(22)		$x = 2m_1$	
			Applying that equation to formula (19) yields:
(23)			$((2m_1)y) = 2k$
			↓ by (3) with "$a := 2,\ b := m_1,\ c := y,\ k := m_1y$"
(24)			true
			That concludes the proof for this case.
		Case 1b:	
(25)		even(y)	
		≡ by (2) with "$z := y$"	
(26)		$(\exists k)(y = 2k)$	
		That formula indicates there exists some value say "n_1" satisfying that quantified formula, i.e.:	
(27)		$y = 2n_1$	
			Applying that equation to formula (19) yields:
(28)			$(x(2n_1)) = 2k$
			≡ by simplifying
(29)			$(2xn_1) = 2\underline{k}$
			Assigning "$k := xn_1$" in that formula yields:
(30)			$(2xn_1) = 2(xn_1)$
			≡ by simplifying
(31)			true
			That concludes the proof for this case.

			Case 2:
(32)			odd(x) ∧ odd(y) → odd(xy)
		We'll assume that formula's antecedents:	
(33)		odd(x)	
		and also:	
(34)		odd(y)	
			and we'll work on proving the consequent:
(35)			odd(xy)
			≡ by (4) with "z := xy"
(36)			(∃k)((xy) = 2k + 1)
			Removing that variable quantification clarifies that to prove that formula, it will suffice to find a value for the variable.
(37)			(xy) = 2k + 1
		Applying the equivalence in formula (4) to formula (33) with "z := x" yields:	
(38)		(∃k)(x = 2k + 1)	
		That formula indicates there exists some value say "n" satisfying that quantified formula, i.e.:	
(39)		x = 2n + 1	
		Applying the equivalence in formula (4) to formula (34) with "z := y" yields:	
(40)		(∃k)(y = 2k + 1)	
		That formula indicates there exists some value say "m" satisfying that quantified formula, i.e.:	
(41)		y = 2m + 1	

			We'll work on transforming the left-hand side of formula (37) to the right-hand side as follows:
(42)			xy
			= by (39)
(43)			(2n+1)y
			= by (41)
(44)			(2n+1)(2m+1)
			= by (5) with "a := 2n, b := 1, c := 2m, d := 1"
(45)			(2n)(2m) + 1(2m) + (2n)•1 + 1•1
			= by simplifying
(46)			4nm + 2m + 2•n + 1
			= by (6)
(47)			(2•2)nm + 2m + 2•n + 1
			= by (3) with "a := 2, b := 2, c := nm"
(48)			[2(2(nm))] + 2m + 2•n + 1
			= by (7) with "a := 2, b := 2(nm), c := m, d := n"
(49)			[2([2(nm)]+m+n)] + 1
			And that satisfies our earlier goal (37) with "k := [2(nm)] + m + n"; i.e. we have:
(50)			true
			That concludes this part of the proof.
			Thus, the theorem that was given is true in all cases.

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