Intersection Between Two Parametric Lines

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Given two line $AB$ and $PQ$, their parametric line equations are

Line$_{AB} : A + s(B - A)$

Line$_{PQ} : P + t(Q - P)$

To find the intersection of the two lines is to find values of $s$ and $t$ that satisfy:

$A + s(B - A) = P + t(Q - P)$

Since $A$, $B$, $P$, and $Q$ are points in two-dimensional space, the above equation is actually two linear equations of two variables $s$ and $t$. If we let $G = B - A$ and $H = Q - P$, the above equations can be written as

$A + sG = P + tH$

$sG - tH = P - A$

Let $C = P - A$

$sG - tH = C$

The 2D coordinates of four points $A$, $B$, $P$, and $Q$ are known, so the $x$- and $y$-component of $G$, $H$, and $C$ can be easily calculated from these four points:

$s g_x - t h_x = c_x$

$s g_y - t h_y = c_y$

or in matrix form:

$\begin{bmatrix} g_x & -h_x \\ g_y & -h_y \end{bmatrix} \begin{bmatrix} s \\ t \end{bmatrix} = \begin{bmatrix} c_x \\ c_y \end{bmatrix}$

Using the Kramer’s method described in class, $s$ and $t$ can be calculated from:

$s = \frac{-c_x h_y + c_y h_x}{-g_x h_y + g_y h_x}$

$t = \frac{g_x c_y - g_y c_x}{-g_x h_y + g_y h_x}$

If $AB$ is the line to be clipped and $PQ$ is one side of the clipping polygon, in Liang-Barsky algorithm we need to calculate only $s$